Self-Corrected Belief-Propagation Decoder for Source Coding with Unknown Source Statistics

Elsa Dupraz, Mohamed Yaoumi

***Abstract*—This paper describes a practical Slepian-Wolf source coding scheme based on Low Density Parity Check (LDPC) codes. It considers the realistic setup where the parameters of the statistical model between the source and the side information are unknown. A novel Self-Corrected Belief-Propagation (SC-BP) algorithm is proposed in order to make the coding scheme robust to incorrect model parameters by introducing some memory inside the LDPC decoder. A Two Dimensional Density Evolution (2D-DE) analysis is then developed to predict the theoretical performance of the SC-BP decoder. Both the 2D-DE analysis and Monte-Carlo simulations confirm the robustness of the SC-BP decoder. The proposed solution allows for an important complexity reduction and shows a performance very close to existing methods which jointly estimate the model parameters and the source sequence.**

***Index Terms*—Slepian-Wolf source coding, LDPC codes, Density Evolution**

I. Introduction

In Slepian-Wolf source coding [1], a source X is compressed, given that a side information Y is only observed by the decoder. If X and Y are statistically dependent, the coding rate is reduced compared to the case without side information. The Slepian-Wolf setup has regained attention recently, due to its application in modern source coding problems such as Distributed Source coding [2], Multi-View video coding [3], or Massive Random Access to data [4].

Most practical Slepian-Wolf source coding schemes rely on channel codes such as Low Density Parity Check (LDPC) codes [3]–[5], and require a precise knowledge of the parameters of the joint probability distribution $P(X,Y)$. To estimate these parameters, solutions based on a feedback link [6] do not cause any rate loss compared to the case where the source parameters are known, but they increase the decoding latency. Alternatively, unknown parameters can be estimated together with the source sequence by using Expectation-Maximization (EM) [7], [8] or Particle Filtering [9] methods at the decoder. However, these methods are penalized by an important complexity since they all require running several times the LDPC decoder. In this paper, we instead propose to run the LDPC decoder once, and to work on improving its robustness against unknown model parameters.

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In the family of LDPC decoders, the Belief Propagation (BP) decoder [10] is the most powerful, but it is not robust against incorrect initial Low Likelihood Ratios (LLR), which are calculated from the joint probability distribution $P(X,Y)$. The Min-Sum (MS) decoder [11] is less efficient than BP, but it shows more robustness against incorrect initial LLR values. The MS decoding performance can be improved by using empirical parameters (scaling, offset, etc.) [11], but their optimal values depend on the parameters of $P(X,Y)$. Alternatively, [12] introduced a Self-Correction (SC) mechanism into the MS decoder. SC compares the signs of messages exchanged in the decoder at successive iterations, and erases messages in cases of sign switching. The subsequent SC-MS decoder does not depend on any empirical parameter and shows a clear performance improvement compared to the MS decoder, while preserving its robustness property. However, SC-MS still shows a small performance loss compared to BP. In this paper, we propose to apply the SC mechanism directly into the BP decoder, hoping that this will improve the robustness of BP against incorrect initial LLRs.

We aim to evaluate the performance of the proposed SC-BP decoder not only from Monte Carlo simulations, but also from a theoretical analysis. The theoretical performance of LDPC decoders is commonly investigated by the Density Evolution (DE) method [10], which evaluates the successive probability distributions $\mathbb{P}(u^{(\ell)})$ of decoder messages $u^{(\ell)}$ at iteration ℓ. However, the SC mechanism introduces some memory inside the decoder, which standard DE fails to capture. This is the reason why, to the best of our knowledge, there does not exist a DE analysis for the original SC-MS decoders. In [13], a generic DE method was proposed for LDPC decoders with memory. But this DE method is very complex, as it requires to evaluate the joint probability distribution $\mathbb{P}(u^{(1)},\cdots,u^{(\ell)})$ of the decoder messages from first to current iteration.

In this paper, we introduce a simpler Two Dimensional (2D) DE analysis that allows to predict the asymptotic performance of SC-MS and SC-BP decoders, by only evaluating the joint probability distribution $\mathbb{P}(u^{(\ell-1)},u^{(\ell)})$ of messages at two successive iterations. Although the proposed 2D-DE analysis is an approximation of the DE method of [13], we show that it accurately predicts the performance of SC-MS and SC-BP decoders. Our 2D-DE analysis then shows that the SC-BP decoder is completely robust against incorrect LLR initialization, unlike the standard BP decoder. Monte Carlo simulations confirm these results, and show that the proposed SC-BP decoder has a performance very close to the joint estimation methods of [7], [8], while allowing for an important complexity reduction compared to these joint methods.

II. Slepian-Wolf source coding

A. Source model

The Slepian-Wolf source coding scheme aims to compress the source X given that the side information Y is only observed by the decoder. We assume that the sources X and Y generate sequences of independent and identically distributed (i.i.d.) symbols $(X_1, \cdots, X_n)$ and $(Y_1, \cdots, Y_n)$, respectively. The source X is binary, with probability distribution given by $p_0 = P(X = 0)$. The side information Y takes its values in an alphabet $\mathcal{Y}$, which is either discrete or continuous. The side information symbols are generated according to a conditional probability distribution $P_\theta(Y|X)$, where θ represents the set of unknown parameters.

The generic model described below can be specified for a large range of distributions. We now describe two examples which will be considered in the simulation section of the paper. In both models, we assume as in [2], [14], that there exists a random variable Z, independent of X, such that $Y = \tilde{X} + Z$, where $\tilde{X} = 1 - 2X$ and $\tilde{X} \in \{-1, 1\}$. Then, in the first model, we consider that Z is Gaussian with mean 0 and unknown variance σ^2, which gives $\theta = \{\sigma^2\}$. In this Gaussian model, the Signal-to-Noise ratio (SNR) can be expressed as SNR $= 1/\sigma^2$. In the second model, we consider that Z is Laplacian with mean 0 and unknown scale parameter λ, which gives $\theta = \{\lambda\}$. In this case, we get SNR $= 1/2\lambda^2$. We consider these two models because they are very common in the main two applications of the Slepian-Wolf setup: the additive Gaussian model is often used in Wireless Sensor Networks applications [2], while the additive Laplacian model is very standard in Distributed Video Coding applications [14]. It is worth noting that the issue of parameter uncertainty was pointed out in both applications, see *e.g.*, [6]–[9].

B. Information Theory results

We now discuss existing information-theoretic results [1], [15], for Slepian-Wolf source coding. Denote by $H_\theta(Y|X)$ the conditional entropy of Y knowing X, for a given θ. If θ is known by both the encoder and the decoder, the minimum achievable rate $R^\star$ needed to transmit X losslessly to the decoder is given by $R^\star = H_\theta(X|Y)$ bits/symbol [1], where $H_\theta(X|Y) \leq H(X)$ whatever the value of θ. On the opposite, if θ is unknown, the minimum achievable rate R is given by $R = \max_\theta H_\theta(X|Y)$ bits/symbol [15], which corresponds to the worst possible case over θ. According to [15], the fact that $R \geq R^\star$ comes from the uncertainty about θ at the encoder. The decoder, on the other hand, can perfectly deal without the knowledge of θ, see [7]–[9].

III. Self-Corrected Belief-Propagation decoder

A. LDPC codes for source coding

Denote by H the binary parity check matrix of size $m \times n$ ($m < n$) of an LDPC code. Assuming that H is full rank, the source coding rate is $R = m/n$. The parity check matrix H can be equivalently represented by a bipartite Tanner graph that connects n Variable Nodes (VN) to m Check Nodes (CN). We use d_v (respectively d_c) to denote the degree of VN $v \in [\![1, n]\!]$ (respectively of CN $c \in [\![1, m]\!]$).

Denote by $\mathbf{x}^n$ the source sequence of length n to compress. A codeword $\mathbf{s}^m$ of length m is then obtained as $\mathbf{s}^m = H\mathbf{x}^n$ [5]. When θ is known, the decoder usually relies on the BP algorithm which aims to recover the source sequence $\mathbf{x}^n$ from the received codeword $\mathbf{s}^m$ and from the side information sequence $\mathbf{y}^n$ [5]. When θ is unknown, existing coding schemes [7]–[9] apply alternating methods that successively estimate the parameter θ and the source sequence $\mathbf{x}^n$ over several iterations. These alternating methods however require running the BP decoder several times. Here, we instead propose a novel LDPC decoder robust to incorrect values of θ.

B. Self-Corrected Belief-Propagation decoder

We now introduce the proposed LDPC SC-BP decoder, which consists of a minor modification of the SC-MS decoder introduced in [12] in the context of channel coding.

In the SC-BP decoder VN messages are initialized as in the BP decoder [8], with Log-Likelihood Ratio (LLR) values

$$u_0 = \log \frac{\mathbb{P}(X=0)\mathbb{P}_\theta(y|X=0)}{\mathbb{P}(X=1)\mathbb{P}_\theta(y|X=1)} \tag{1}$$

The expression in (1) depends on the parameter value θ. For instance, for the Gaussian model described in Section II, $u_0 = \log \frac{p_0}{1-p_0} + \alpha y$, where $\alpha = 2/\sigma^2$. And for the Laplacian model, $u_0 = \log \frac{p_0}{1-p_0} + \alpha(|y+1| - |y-1|)$, where $\alpha = 1/\lambda$. If the parameters σ^2 or λ are unknown, we set α to arbitrary values and we say that the decoder is mismatched.

We then consider that the decoder works in a maximum of L iterations. At each iteration $\ell \in [\![1, L]\!]$, the $(d_c - 1)$ incoming messages to a CN of degree d_c are denoted $\mathbf{t}^{(\ell)} = (t_1^{(\ell)}, \cdots, t_{d_c-1}^{(\ell)})$, and the CN mapping Φ_c has expression

$$\Phi_c(\mathbf{t}^{(\ell)}) = (1-2s)\left(\left(1 + \prod_{v=1}^{d_c-1} \tanh \frac{t_v^{(\ell)}}{2}\right) / \left(1 - \prod_{v=1}^{d_c-1} \tanh \frac{t_v^{(\ell)}}{2}\right)\right) \tag{2}$$

where s is the codeword bit value at the CN. Equation (2) corresponds to the standard CN mapping for the BP decoder applied to Slepian-Wolf source coding [5].

Then, incoming messages to a VN of degree d_v are denoted $\mathbf{u}^{(\ell)} = (u_0, u_1^{(\ell)}, \cdots, u_{d_v-1}^{(\ell)})$, where u_0 is given in (1). Output VN messages are calculated in two steps. We first compute candidate messages $x^{(\ell+1)} = \Phi_v(\mathbf{u}^{(\ell)})$ from a VN mapping Φ_v given by

$$\Phi_v(\mathbf{u}^{(\ell)}) = u_0 + \sum_{c=1}^{d_v-1} u_c^{(\ell)}. \tag{3}$$

We then apply the SC mapping Φ_{sc} defined as

$$\Phi_{sc}(x^{(\ell)}, x^{(\ell+1)}) = \begin{cases} 0 & \text{if } \mathrm{sign}(x^{(\ell+1)}) \neq \mathrm{sign}(t^{(\ell)}) \\ x^{(\ell+1)} & \text{otherwise.} \end{cases} \tag{4}$$

In the above expression, $t^{(\ell)} = \Phi_{sc}(x^{(\ell-1)}, x^{(\ell)})$ is the output of the SC mapping at previous iteration. By convention, the value 0 is assumed to be both positive and negative, which means that if $t^{(\ell)} = 0$, then $t^{(\ell+1)} = x^{(\ell+1)}$. In addition, the SC mapping (4) is only applied from iteration 2. At iteration 1, we always set $t^{(1)} = x^{(1)}$. The SC mapping (4) constitutes the main difference between the standard BP decoder and the proposed SC-BP decoder, although the SC mapping (4) was previously introduced for a MS decoder in [12].

IV. Density Evolution analysis

DE allows to predict the performance of an LDPC decoder under the assumption that the codeword length tends to infinity [10]. DE is often used to theoretically evaluate the performance of LDPC decoders, as it allows to both validate empirical results observed from Monte Carlo simulations, and to optimize the code and decoder parameters in an efficient way [10]. Standard DE [10] iteratively calculates marginal probability distributions $\mathbb{P}(u^{(\ell)})$ of messages exchanged in the decoder. However, evaluating only the marginal probability distributions may not be accurate for SC-MS and SC-BP decoders, because the SC mapping (4) introduces some statistical dependencies between messages at successive iterations. A general DE method was proposed in [13] for LDPC decoders with memory, but this general method needs to be specified for particular LDPC decoders. As examples, [13] only considers LDPC decoders with large memory, where *e.g.*, messages at iteration ℓ were calculated from all previous messages from iteration 1 to $\ell - 1$. For these complex decoders, the DE method of [13] requires to express the successive joint probability distributions $\mathbb{P}(u^{(1)}, \cdots, u^{(\ell)})$ of messages from the first iteration to the current iteration ℓ. This makes the analysis intractable, since this requires to evaluate probability distributions of vectors of length ℓ.

In this part, we propose a 2D-DE analysis for the SC-BP decoder, by only calculating the joint probability distributions $\mathbb{P}(u^{(\ell)}, u^{(\ell+1)})$ of messages at successive iterations. The introduced 2D-DE analysis is a tractable approximation of the general DE method of [13]. In this paper, the 2D-DE analysis is described for the SC-BP decoder, but it can be easily applied to the SC-MS decoder, by considering the MS mapping instead of the BP mapping in (2).

A. DE assumptions

In order to obtain tractable DE expressions, we consider that the messages exchanged in the decoder are quantized on q bits, where q is large enough so as to ensure that the performance of the quantized decoder is very close to the performance of the infinite-precision decoder [10]. In addition, we consider the main two assumptions of standard DE [10], [13]: (i) since we consider an asymptotic codeword length, we assume that the code is cycle-free, (ii) since all the VN, CN, SC mappings involved in the SC-BP decoder are symmetric, we consider that the all-zero codeword was transmitted.

B. 2D-DE equations

We now provide the iterative 2D-DE equations for the SC-BP decoder. At iteration ℓ, the joint probability distribution $\mathbb{P}(u^{(\ell)}, u^{(\ell+1)})$ of CN output messages is evaluated as

$$\mathbb{P}(u^{(\ell)}, u^{(\ell+1)}) = \sum_{\substack{\mathbf{t}^{(\ell)}, \mathbf{t}^{(\ell+1)}: \\ \Phi_c(\mathbf{t}^{(\ell)}) = u^{(\ell)} \\ \Phi_c(\mathbf{t}^{(\ell+1)}) = u^{(\ell+1)}}} \prod_{v=1}^{d_c-1} \mathbb{P}(t_v^{(\ell)}, t_v^{(\ell+1)})$$

where Φ_c is the CN mapping in (2) and $\mathbb{P}(t_v^{(\ell)}, t_v^{(\ell+1)})$ is the joint probability distribution of CN input messages $t_v^{(\ell)}, t_v^{(\ell+1)}$. Then, the joint probability distribution $\mathbb{P}(x^{(\ell)}, x^{(\ell+1)})$ of VN output messages is given by

$$\mathbb{P}(x^{(\ell)}, x^{(\ell+1)}) = \sum_{\substack{\mathbf{u}^{(\ell)}, \mathbf{u}^{(\ell+1)}: \\ \Phi_c(\mathbf{u}^{(\ell)}) = x^{(\ell)} \\ \Phi_c(\mathbf{u}^{(\ell+1)}) = x^{(\ell+1)}}} \mathbb{P}(u_0) \prod_{c=1}^{d_v-1} \mathbb{P}(u_c^{(\ell)}, u_c^{(\ell+1)})$$

where Φ_v is the VN mapping in (3). Finally, the joint probability distribution $\mathbb{P}(t^{(\ell)}, t^{(\ell+1)})$ of SC output messages is given by

$$\mathbb{P}(t^{(\ell)}, t^{(\ell+1)}) = \sum_{\substack{x^{(\ell-2)}, x^{(\ell-1)}, x^{(\ell)}: \\ \Phi_{sc}(x^{(\ell-2)}, x^{(\ell-1)}) = t^{(\ell)} \\ \Phi_{sc}(x^{(\ell-1)}, x^{(\ell)}) = t^{(\ell+1)}}} \mathbb{P}(x^{(\ell-2)}, x^{(\ell-1)}, x^{(\ell)}).$$

where the SC mapping is given in (4). We then perform the following simplification, in order to tag along with a 2D analysis:

$$\begin{aligned}\mathbb{P}(x^{(\ell-2)}, x^{(\ell-1)}, x^{(\ell)}) &= \mathbb{P}(x^{(\ell-2)}, x^{(\ell-1)}) \mathbb{P}(x^{(\ell)} | x^{(\ell-1)}, x^{(\ell-2)}) \\ &\approx \mathbb{P}(x^{(\ell-2)}, x^{(\ell-1)}) \mathbb{P}(x^{(\ell)} | x^{(\ell-1)}), \qquad (5)\end{aligned}$$

and we calculate the conditional probability $\mathbb{P}(x^{(\ell)} | x^{(\ell-1)})$ from the joint distribution $\mathbb{P}(x^{(\ell-1)}, x^{(\ell)})$. Finally, the decoder error probability at iteration ℓ is evaluated from $\mathbb{P}(x^{(\ell-1)}, x^{(\ell)})$ as

$$P_e^{(\ell)} = \sum_{x^{(\ell-1)}} \left(\frac{1}{2} \mathbb{P}(x^{(\ell-1)}, 0) + \sum_{x^{(\ell)} < 0} \mathbb{P}(x^{(\ell-1)}, x^{(\ell)}) \right).$$

In order to obtain the approximation in (5), we considered that $\mathbb{P}(x^{(\ell)} | x^{(\ell-1)}, x^{(\ell-2)}) \approx \mathbb{P}(x^{(\ell)} | x^{(\ell-1)})$ by assuming the following Markov chain: $x^{(\ell-2)} \to x^{(\ell-1)} \to x^{(\ell)}$ between the VN outputs at successive iterations. This assumption allows to greatly simplify the DE analysis while still capturing the SC relation between $x^{(\ell-1)}$ and $x^{(\ell)}$. In the next section, we numerically evaluate the accuracy of the proposed 2D-DE, and show that considering only the dependency with messages from iteration $\ell - 1$ is sufficient to accurately predict the performance of the SC-BP decoder.

V. Numerical results

A. DE analysis

In this section, we consider the additive Gaussian model described in Section II-A, and four regular (d_v, d_c)-codes given in Table I. We use DE to evaluate the theoretical performance of five LDPC decoders: BP, MS, offset MS, SC-MS, and the proposed SC-BP decoder. In the DE analysis, we consider $L = 100$ iterations, and $q = 6$ quantization bits. For each considered code and decoder, we evaluate the *threshold*

Code	BP	BP mis.	MS	MS mis.	MS optim.	MS optim. mis.	SC-MS	SC-MS mis.	SC-BP	SC-BP mis.
(3,4)	1.12	1.52	2.12	2.09	1.16	2.87	1.16	1.14	1.04	1.08
(3,5)	0.99	1.27	1.71	1.64	1.05	2.18	1.02	1.02	0.95	0.97
(3,6)	1.20	1.38	1.74	1.68	1.23	2.12	1.21	1.21	1.16	1.18
(4,5)	2.74	2.82	4.00	4.03	2.74	2.90	2.75	2.75	2.59	2.57

TABLE I

THRESHOLD COMPARISON OF SEVERAL LDPC DECODERS UNDER THE GAUSSIAN MODEL WITH AND WITHOUT MISMATCH. ALL THE THRESHOLD VALUES ARE IN dB.

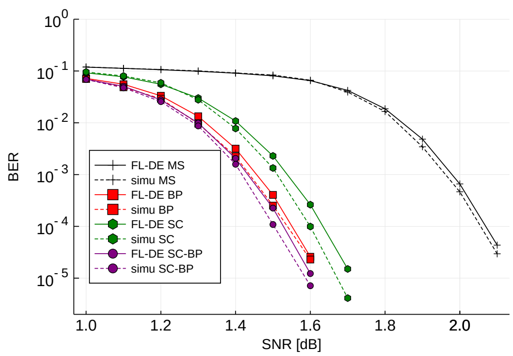

Fig. 1. Comparison between BER performance predicted by FL-DE method of [16] and between decoder performance evaluated from Monte Carlo simulations, for the $(3,6)$-code under the Gaussian model

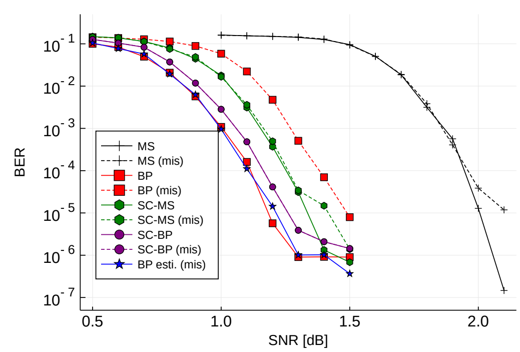

Fig. 2. BER performance of the irregular code of length $N = 8192$, for different LDPC decoders under the Gaussian model, with and without mismatch. The curves for SC-BP with and without mismatch are superimposed.

as the minimum SNR value for which the final decoder error probability $P_e^{(L)} < 10^{-6}$. The decoder error probabilities are calculated from standard DE [10] for the BP, MS, and offset MS decoders, and from the 2D-DE of Section IV for SC-MS and SC-BP decoders.

Table I provides the thresholds obtained when the decoders are initialized with the correct SNR value, and the thresholds in the mismatched setup where the decoders are always initialized with an arbitrary value $\alpha = 4$, see Section III-B. In particular, for the offset-MS decoder, the offset parameter is optimized from this mismatched SNR value. We first observe that in the standard setup without mismatch, the decoders show threshold values very close to each other, except for the MS decoder which has a degraded performance. Then, in the mismatched setup, the BP decoder and the offset MS decoder show an important performance degradation compared to the case without mismatch. On the opposite, the thresholds for MS, SC-MS, and SC-BP decoders remain approximately the same, which confirms their robustness in the mismatched setup. Finally, the SC-BP decoder has the best threshold values.

B. FL-DE analysis

We now verify the accuracy of the 2D-DE analysis introduced in Section IV. For this, we rely on the Finite-Length Density Evolution (FL-DE) method proposed in [16]. This method allows to estimate the finite-length performance of an LDPC decoder, from a formula that depends on the asymptotic error probabilities provided by DE. Although this method does not take into account the effect of cycles, [16], [17], show that it accurately predicts the decoder performance measured from Monte Carlo simulations, for long enough codewords. The formula proposed in [16] can be straightforwardly used in our case, by using in the formula the asymptotic error probabilities provided by 2D-DE. With the Gaussian model, we consider a regular $(3,6)$-code and a codeword length $N = 10^4$. We focus on four LDPC decoders: BP, MS, SC-MS, SC-BP, initialized with the correct SNR values. For each decoder, we calculate the Bit Error Rate (BER) predicted by the FL-DE method of [16], applied either with standard DE (for BP and MS decoders), or with the proposed 2D-DE analysis (for SC-MS and SC-BP decoders).

Figure 1 provides the BERs predicted by the FL-DE method, and the BERs measured from Monte Carlo simulations. For BP and MS decoders, we see a gap of less than 0.01dB between the two curves, while for SC-BP and SC-MS, we see a gap of less than 0.05dB. These results confirm that the proposed 2D-DE analysis can be used to predict the performance of the SC-BP and SC-MS decoders. We also see that, in accordance with the threshold values of Table I, the proposed SC-BP decoder shows a slight performance gain compared to the standard BP decoder.

C. Monte Carlo simulations

We now evaluate the performance of the SC-BP decoder from Monte Carlo simulations, for the Gaussian and Laplacian models described in Section II-A. For the Gaussian model, we set $L = 50$ iterations, and we consider an irregular, protograph-based LDPC code of length $N = 8192$, with protograph S_0 optimized in [17] for good performance over a Gaussian channel. We consider four LDPC decoders: MS, BP, SC-MS, and SC-BP, as well as the method of [7] which jointly estimates the unknown parameter θ and the codeword. For this method, we set $K = 3$ steps of joint estimation. Figure 2 represents the BER with respect to SNR, for the Gaussian model. For each considered decoder, we show the results for the standard setup where the decoder knows the SNR value,

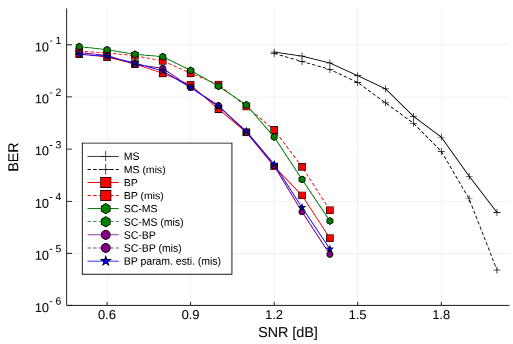

Fig. 3. BER performance of (3,6)-code for different LDPC decoders under the Laplacian model, with and without mismatch. The curves for SC-BP with and without mismatch are superimposed.

and for the mismatched setup in which the decoder is always initialized with arbitrary value $\alpha = 4$. We observe that the BP decoder shows an important performance degradation in the mismatched setup. On the opposite, the SC-BP decoder remarkably does not suffer from any performance loss in the mismatched setup, and the SC-MS decoder is also robust to mismatched parameters, as already observed in [12]. At the end, the SC-BP decoder shows a performance very close to the method of [7]. Finally, Figure 3 represents the BER with respect to SNR, for the Laplacian model, for a regular $(3, 6)$-code of length $N = 10^4$. In the mismatched setup, the decoder is initialized with arbitrary value $\alpha = 4$. In this case, we observe that the SC-BP decoder has the same performance as the method of [7].

D. Complexity Analysis

We now compare the complexity of four of the previously considered decoding solutions: BP, SC-BP, SC-MS, and BP with parameter estimation [7]. Since these decoders have approximately the same CN complexity, we only compare the complexity of the VN and SC operations. According to [17], the VN mapping (3) requires $2d_v$ additions. In addition, the SC mapping (4) only consists of d_v comparisons (one per message). Therefore, BP requires $2d_v\tilde{L}$ VN operations, while SC-BP and SC-MS need $3d_v\tilde{L}$ VN operations, where $\tilde{L} \leq L$ is the average number of iterations measured from simulations (with a stopping criterion in the decoder). Then, since the solution of [7] runs the BP decoder K times, the total number of VN operations is given by $2d_v \sum_{k=1}^{K} \tilde{L}_k$, where $\tilde{L}_k$ is the average number of iterations at the k-th run.

We now numerically evaluate the number of VN operations with the Gaussian model, by measuring the average number of iterations $\tilde{L}$ and $\tilde{L}_k$ from Monte Carlo simulations. For the regular $(3, 6)$-code and with the same decoder parameters as in Section V-C, at SNR$=1.5$dB, we get 167 operations for the BP decoder, 221 for SC-BP, 236 for SC-MS, and 407 for BP with parameter estimation [7]. For the irregular LDPC code considered in Figure 2, at SNR=1dB, and by replacing d_v by the average VN degree in the above expressions, we get 322 operations for BP decoder, 371 for SC-BP decoder, and 750 for the solution of [7]. This shows the important complexity reduction raised by the proposed SC-BP decoder.

VI. Conclusion

In this paper, we introduced a novel SC-BP LDPC decoder for Slepian-Wolf source coding. We developed a 2D-DE analysis for performance prediction of SC-based decoders. Both the 2D-DE analysis and the Monte Carlo simulations confirm that the proposed SC-BP decoder outperforms existing BP and SC-MS decoders in the mismatched setup. They also show that SC-BP has the same performance has existing joint estimation-decoding method, while allowing for a clear complexity reduction.

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